CIMPA SCHOOL

NONCOMMUTATIVE GEOMETRY AND INDEX THEORY

CIMAT MÉRIDA

December 3rd-15th, 2018

ORGANIZERS

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Centro de Ciencias Matemáticas, UNAM

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SCHEDULE First Week: 3-7 december 2018

	Monday	Tuesday	Wednesday	Thursday	Friday
9h00-9h40	Transportation	Transportation	Transportation	Transportation	Transportation
10h00-11h30	**Skandalis 1**	**Skandalis 2**	**Skandalis 3**	**Skandalis 4**	**Wang 4**
11h30-12h00	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
12h00-13h30	**Wang 1**	**Wang 2**	**Vega** 12h00-13h00	**Wang 3**	**Angel** 12h00-12h40 **Díaz** 12h50-13h30
13h30-15h00	Lunch	Lunch	Lunch 13h00-14h00	Lunch	Lunch
15h00-15h40	**Belcher**	**Kordon**	Transportation Dzibilchaltún 14h00-14h30	**Ruiz-Castrillón**	Transportation to hotel
15h40-16h20	Problems Interaction	Problems Interaction	Visit Dzibilchaltún 14h30-17h00	Problems Interaction 15:40-17:10	
16h20-17h50	**Carrillo-Rouse**	**Vassout**	Transportation to UADY (17h00-17h15) Colloquium UADY **Bárcenas** (17h30-18h30)	**Franco** 17h10-17h50	
18h00-18h40	Transportation to hotel	Transportation to hotel	Transportation to hotel (18h30-19h00)	Transportation to hotel	

SCHEDULE Second Week: 10-14 december 2018

	Monday	Tuesday	Wednesday	Thursday	Friday
9h00-9h40	Transportation	Transportation	Transportation	Transportation	Transportation
10h00-11h30	**Debord 1**	**Joachim 1**	**Lescure 3**	**Schick 3**	**Schick 4**
11h30-12h00	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
12h00-13h30	**Lescure 1**	**Schick 2**	**Debord 3**	**Joachim 3**	**Joachim 4**
13h30-15h00	Lunch	Lunch 13h30-14h40	Lunch 13h30-14h50	Lunch	Lunch
15h00-15h40	**López-Castaño**	**Lescure 2** 14h40-16h10	**Joachim 2** 14h50-16h20	**Mohsen**	Transportation to hotel
15h40-16h20	Problems Interaction	Break 16h10-16h20	Transportation to hotel 16h30-17h10	Problems Interaction	
16h20-17h50	**Schick 1**	**Debord 2**		**Lescure 4**	
18h00-18h40	Transportation to hotel	Transportation to hotel		Transportation to hotel	
20h-22h00 22h00-22h30			Dinner CHAYA MAYA Transportation to hotel		

COURSES

Claire Debord, Université Blaise Pascal, France.
Algebroids and Groupoids in Index theory and differential geometry.

Jean Marie Lescure, Université Blaise Pascal, France.
Geometric description of operators via noncommutative geometry, applications to index theory for singular manifolds.

Georges Skandalis, Université Paris Diderot, France.
An introduction to noncommutative geometry.

1. The Hilbert space, adjoints, spectrum, unbounded operators.
2. C^*-algebras (definitions and some examples).
3. K-theory of C^*-algebras - main properties, exact sequences, Bott periodicity and some computations.
4. Kasparov's KK-theory (If time permits)

Michael Joachim, Universität Münster, Germany.
Index Theory and Positive scalar curvature.

Hang Wang, East China Normal University, China.
Index theory for equivariant elliptic operators on homogeneous spaces.

Abstract: We introduce index theory of G-equivariant elliptic operators on a homogeneous space of the form G/H, where G is a Lie group and H is a compact subgroup, and show some nice links to the unitary representations of G, either in the square-integrable functions of G or in a compact quotient by a discrete subgroup. A tentative timeline is the following.

Lecture 1: We define elliptic operators and introduce their indices in K-theory, and in the special case of G being unimodular, a numerical index named L^2-index.
Lecture 2: We use examples of semisimple Lie groups to display some relation between (tempered) unitary representations of the groups and their C^*-algebras and K-theory.
Lecture 3: For a semisimple Lie group with discrete points in its tempered dual, the L^2-index of an elliptic operator admits a cohomological formula by Connes-Moscovici and also a formula by Atiyah-Schmid calculating the formal dimension of some distinct class of representations of the group.
Lecture 4: If G has a discrete subgroup Γ with compact quotient, there is a nice relation between indices in K-theory of C^*-algebras of G and that of Γ, which links to an index theoretic interpretation of the Selberg trace formula.

Thomas Schick, Universität Göttingen, Germany.
Coarse Index Theory.

TRAINING SESSIONS

Paulo Carrillo Rouse, Institut de Mathématiques de Toulouse, France.
Title: **Introduction to smooth manifolds.**

Stéphane Vassout, Université Paris Diderot, France.
Title: **Introduction to pseudodifferential operators.**

CONTRIBUTED TALKS

Jonathan Belcher, University of Colorado at Boulder, USA.
Title: **Bridge cohomology: A generalization of Hochschild and cyclic cohomologies with applications to Chern-Weyl theory.**

Abstract: The connection between Hochschild and cyclic cohomologies with generalized De Rham homology and index theories for arbitrary algebras has long been established by the work of Connes, Karoubi,

Loday, Feigin, Tsygan, et al. Here we generalize these cohomology theories even further, essentially creating a theory that establishes a step-wise bridge between the two. Motivation for this construction comes from trying to generalize the Hochschild-Kostant-Rosenberg-Connes theorem to manifolds with boundary, and applications in tracial constructions in certain classes of pseudo-differential operators. Further geometric and topological interests of this theory include extending Chern-Weil theory to manifolds with boundary via pairings between bridge cohomology and higher K-theories.

Francisco Kordon, Universidad de Buenos Aires, Argentina.
Title: **The Hochschild cohomology of the enveloping algebra of a Lie-Rinehart pair.**

Abstract: Let us fix a ground field k. A Lie-Rinehart pair (S, L) consists of a commutative algebra S and a Lie algebra L with an S-module structure that acts on S by derivations and which satisfies certain compatibility conditions. An important example is the pair $(S, \mathrm{Der}S)$ with second component the Lie algebra of derivations of S. The universal enveloping algebra of a pair (S, L) is the smaller associative algebra that admits a morphism of algebras $S \to U$ and a morphism of Lie algebras $L \to U$. If M is a U-module, the Lie-Rinehart cohomology of the pair with values on M was defined by G. Rinehart as $H^*(L|S, M) = \mathrm{Ext}_U^*(S, M)$. This generalizes the usual Lie algebra cohomology of L by taking into account its interaction with S. We are interested in computing the Hochschild cohomology $HH^*(U)$ of the enveloping algebra U. Our main result is the construction of a spectral sequence converging to it, that is based on ideas by Th. Lambre and P. Le Meur.

Theorem. If L is S-projective then there is a first-quadrant spectral sequence E^* converging to $HH^*(U)$ with second page $E_2^{p,q} = H^p(L|S, H^q(S, U))$.

In particular, there is a U-module structure on $H^*(S, U)$, the Hochschild cohomology of S with values on U. We construct it using an U^e-injective resolution of U and, later, following ideas by M. Suárez-álvarez, provide an alternative realization in terms of an S^e-projective resolution of S, which is needed for explicit computations.

In this talk, we will go over the construction of the spectral sequence, we will see some immediate consequences and we will illustrate the fact that the spectral sequence makes it easier to compute the Hochschild cohomology of U in some examples. This work is part of the ongoing PhD research of the author.

Javier Vega, Universidad Autónoma de Nuevo León, Mexico.
Title: **Mathematical foundations of physics. A language for its unified treatment.**

Abstract: Mathematics has result to provide an excellent framework for description of physical phenomena. A panoramic exposition of the main results of classical physics in terms of Clifford Algebra is presented. As well, links with Symplectic Geometry of Mechanics are discussed. Finally, process of Geometric Quantization is sketched and its attributes as a possible foundation for Quantum Field Theory reviewed.

Noé Bárcenas, Centro de Ciencias Matemáticas, UNAM Morelia, Mexico.
Title: **Topological Rigidity of Alexandrov Spaces**.

Abstract: We will discuss several conjectures generalizing the Borel Conjecture for aspherical manifolds to the context of metric geodesic spaces with a sinthetic notion of non-negative curvature, alexandrov spaces. We will show how classiffication efforts via group actions, Gromov - Hausdorff convergence and ideas from Geometric Group Theory give evidence for these conjectures by proving special cases.

Carlos Luis Franco, Universidade de Santiago de Compostela, Spain.
Title: **Atiyah-Singer Index Theorem for Dirac Operators.**

Abstract: I present the proof of the Atiyah-Singer index theorem for Dirac operators using physical ideas, like the Schwartz kernel asymptotic expansion of the heat equation and the Getzler symbolic calculus. This theorem concerns compact, even-dimensional, oriented Riemannian manifolds without boundary, and equipped with Clifford bundles. The corresponding Dirac operator acts on its smooth sections, mapping even sections to odd sections with respect to a grading of the Clifford bundle. It is an elliptic operator, and thus it is Fredholm in the sense that that its kernel and cokernel are finite dimensional. So the analytical index can be defined as the difference between these two dimensions. The index theorem gives an equality between this analytical index and the evaluation of a certain characteristic class, the Â-genus, in the funda-

mental homology class of the manifold (integral of the $\hat{A}$-genus over the manifold). Considering arbitrary coefficients, the index theorem is very general, including the Gauss-Bonnet, signature and Riemann-Roch theorems, and it is useful in Geometry, Topology and Theoretical Physics. The proof involves a wide variety of different concepts, like curvature, characteristic classes, Clifford algebras, Dirac operators, spin structures, Sobolev spaces, spectral decomposition, Hodge decomposition, trace-class operators, Schwartz kernels, asymptotic expansions and symbolic calculus.

Juan Felipe Ruiz Castrillón, Universidad Nacional de Colombia, Colombia.
Title: **A categorical approach to several Galois theories.**

Abstract: In this talk we discuss the concept of Galois structure and Galois epimorphism in a general setting. Namely, a Galois structure for an epimorphism $\pi\colon M \to B$ in some category Cat is the action of a group object that gives to M the structure of principal homogeneous space in the relative category CatB. We see that this general setting applies to classic Galois theory, coverings and fibrations with G-invariant connections.

Andrés Ángel, Universidad del Norte, Colombia.
Title: **$\mathbb{Z}_k$-stratifolds.** (Joint work with Carlos Segovia, Fernando Torres)

Abstract: $\mathbb{Z}_k$-manifolds were introduced by Sullivan and are useful objects in the study of index problems (with $\mathbb{Z}_k$)-coefficients. Kreck introduced stratifolds to represent homology classes with integer coefficients. Generalizing ideas of Sullivan and Kreck we define $\mathbb{Z}_k$-stratifolds. We show that those objects carry fundamental classes in homology with $\mathbb{Z}_k$-coefficients and define a homology theory isomorphic to singular homology with $\mathbb{Z}_k$-coefficients. We use this machinery to study the filtration induced by the Atiyah-Hirzebruch spectral sequence of $\mathbb{Z}_k$-bordism (k odd) and to give geometric constructions of the homology classes of $B\mathbb{Z}_p \times B\mathbb{Z}_p$ which aim to understand the geometric nature of non-representable classes in the sense of Steenrod.

Fredy Díaz García, CCM-UNAM Morelia / UMSNH, Mexico.
Title: **A spectral triple for noncommutative compact surfaces.**

Abstract: The aim of the talk is to present the construction of a spectral triple for noncommuta- tive compact surfaces of any genus given by subalgebras of the Toeplitz algebra. Connes' axioms for noncommutative spin geometries will be analized for these examples. It will be shown that the regularity and finiteness conditions hold. The existence of a real structure can only be proven under a certain modification of the axioms. For instance, the first order condition holds up to compacts of arbitrary small order. On the other hand, the orientation condition and the Poincaré duality fail.

Juan Daniel López Castaño, Universidad Nacional de Colombia, Colombia.
Title: **Spectral Geometry and Spectral Action.**

Abstract: Strongly motivated by physics (relativity and quantum mechanics), Connes and Chamseddine have defined the spectral action based on spectral facts. The goal of this talk is to review the necessary tools of noncommutative geometry and its spectral approach (spectral geometry) which are behind the spectral action to be able to compute it on few examples. In that way, first, I will present standard material of noncommutative integration theory around the notion of spectral triple (A, H, D). This means to understand the notion of differential (or pseudodifferential) operators in this context. Next, I will explain the fundamentals of heat kernel theory and its expansion as $t \to 0^+$ in terms of coefficients of the elliptic generalised laplacian operator Δ, with a method to compute the coefficients of this expansion. This coefficients will be linked, via the noncommutative integrals of powers of $|D|$, with the asymptotic expansion in Λ of the spectral action $\mathrm{Tr}(f(D/\Lambda))$. This action plays an essential role in physics, and, during the talk, I will relate it with the Einstein-Hilbert action in gravity and the Yang-Mills action in particle physics. For each part of the talk, I will suggest references since this one is by no means original.

Omar Mohsen, University Paris Diderot, France.
Title: **The inhomogeneous pseudodifferential calculus.**

Abstract: In 1974, Folland and Stein constructed an inhomogeneous pseudo-differential calculus based on analysis on the Heisenberg group. This Heisenberg calculus was generalized by several authors, to any

subbundle of the tangent bundle. Recently, van Erp and Yuncken, following Debord and Skandalis showed that this calculus can be recovered using a deformation groupoid à la tangent groupoid of Connes.Using functoriality of the deformation to the normal cone construction, we give an elementary construction of this groupoid. We then extend it to the general case of a filtration of the tangent bundle by an iterated deformation.